# Lesson 5-3: Concurrent Lines, Medians and Altitudes

## Playing with bisectors

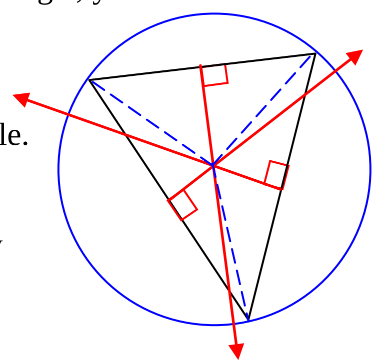
Yesterday we learned some properties of perpendicular bisectors of the sides of triangles, and of triangle angle bisectors. Today we are going to use those skills to construct special sets of lines related to triangles and explore their properties.

## Center of a triangle

If I asked you to find the center of a circle, I'm sure you could do that easily. But, what if I asked you to find the center of a triangle? Hmm, what does that even mean? Let's see…

## Perpendicular bisectors

If you construct all three perpendicular bisectors for a given triangle, you will notice a few things:



1. They intersect in one point.
2. That point is equidistant from the vertices of the triangle.

Now, if you have one point, and a set of points that are equidistant from that point, what figure does that make? They make a circle. In this case the circle contains the triangle and intersects all three vertices.

OK, we have some very interesting properties here. Let's define some terms to make it easier to talk about this.

## Concurrent lines

Three or more lines that intersect at one point are said to be ***concurrent lines***.

## Point of concurrency

The point that concurrent lines meet at is called the ***point of concurrency***.

## Interesting points of concurrency

There are four points of concurrency, or triangle centers that we will look at:

1. Perpendicular bisectors
2. Angle bisectors
3. Medians
4. Altitudes

Now, in case you don't know what medians or altitudes are:

A ***median of a triangle*** is a segment whose endpoints are a vertex and the midpoint of the opposite side.

An ***altitude of a triangle*** is the perpendicular segment from a vertex to the line containing the opposite side. An altitude may be on a side of the triangle or outside the triangle.

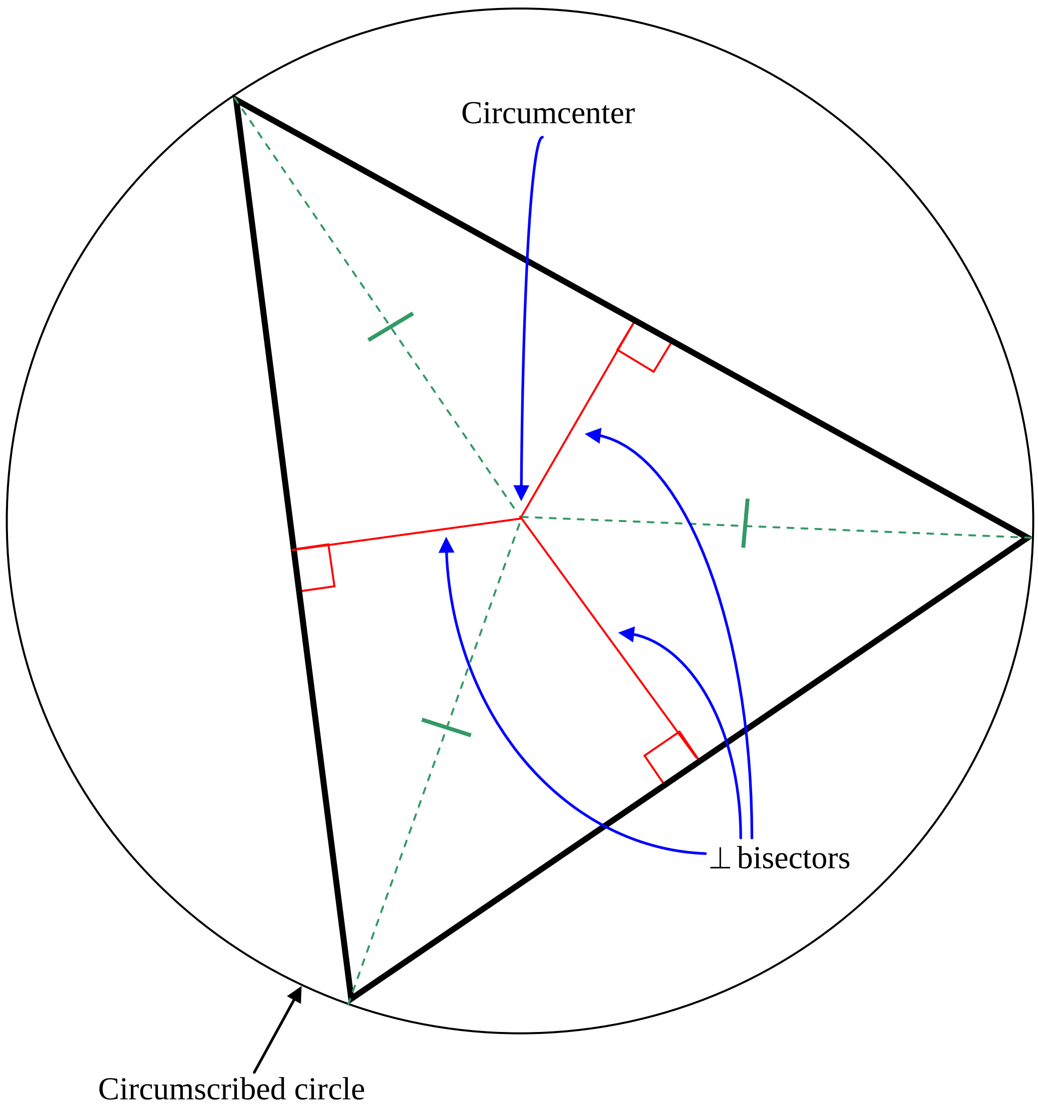
# Lesson 5-3: Concurrent Lines, Medians and Altitudes

## Perpendicular bisectors of a triangle – "Circumcenter"

The point of concurrency of the perpendicular bisectors is equidistant from the triangle vertices and hence is the center of a circle that contains the vertices.

This circle is said to be ***circumscribed*** about the triangle.

The point of concurrency of the perpendicular bisectors is also called the ***circumcenter*** of the triangle.




## Theorem 5-6

The perpendicular bisectors of the sides of a triangle are concurrent at a point equidistant from the vertices.

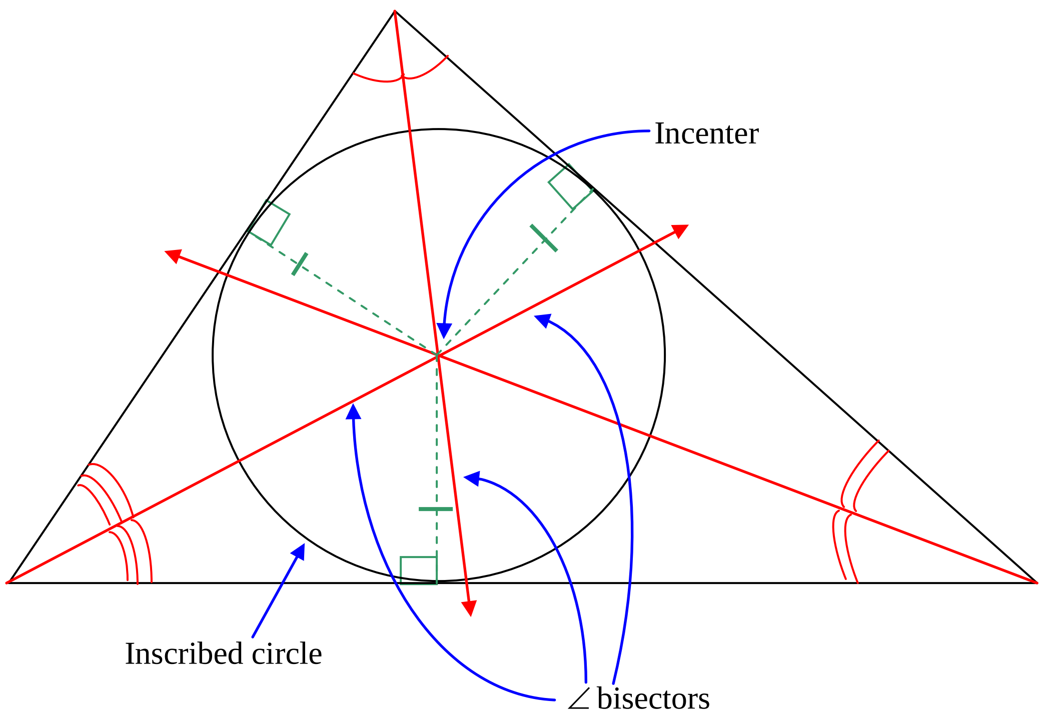
# Lesson 5-3: Concurrent Lines, Medians and Altitudes

## Angle bisectors of a triangle – "Incenter"

The point of concurrency of the angle bisectors is equidistant from the sides and hence is the center of a circle that contains points on the sides of the triangle.

The circle is said to be ***inscribed*** in the triangle.

The point of concurrency of the angle bisectors is also called the ***incenter*** of the triangle.




## Theorem 5-7

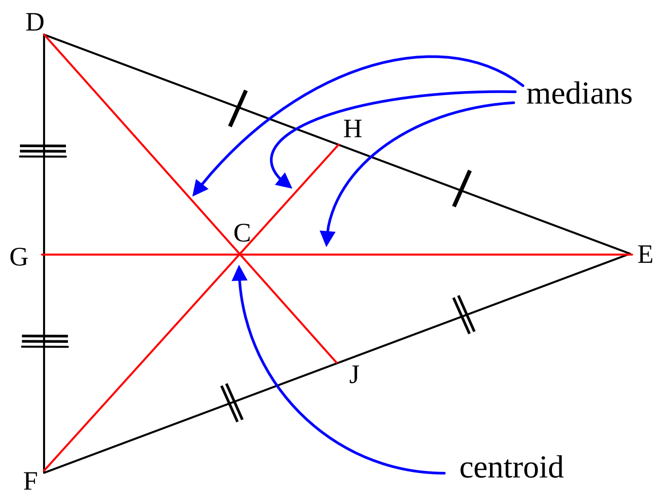
The bisectors of the angles of a triangle are concurrent at a point equidistant from the sides.

# Lesson 5-3: Concurrent Lines, Medians and Altitudes

## Medians of a triangle – "Centroid"

The point of concurrency of the medians of a triangle is called the center of gravity of the triangle. This is the point on which the triangle would balance. It is the triangle's center of mass.

The point of concurrency of the medians is also called the ***centroid***.




The centroid is $\frac{2}{3}$ the distance along the median from the vertex.

## Theorem 5-8

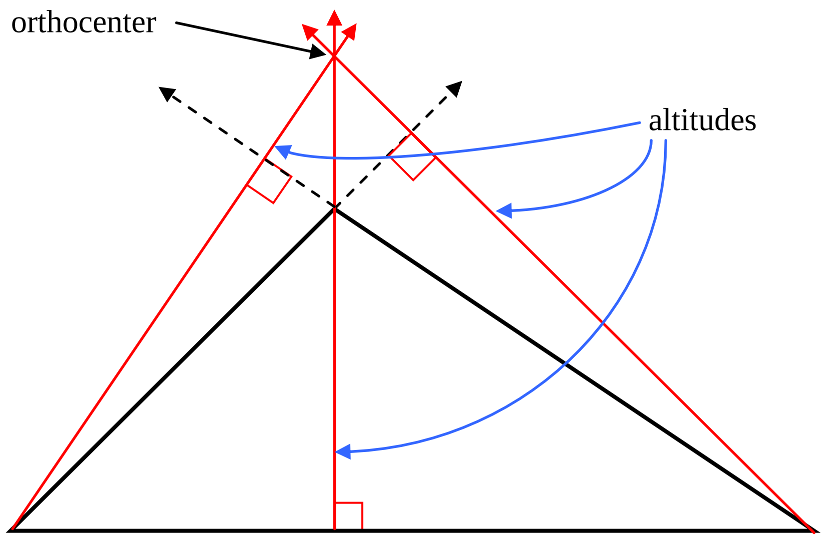
The medians of a triangle are concurrent at a point that is two thirds the distance from each vertex to the midpoint of the opposite side.

$$DC = \frac{2}{3}DJ \qquad EC = \frac{2}{3}EG \qquad FC = \frac{2}{3}FH$$

# Lesson 5-3: Concurrent Lines, Medians and Altitudes

## Altitudes of a triangle – "Orthocenter"

The point of concurrency for the altitudes of a triangle is called the ***orthocenter*** of the triangle.

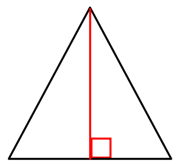



## Theorem 5-9

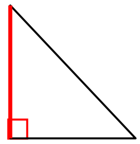
The lines that contain the altitudes of a triangle are concurrent.

## Location of altitudes

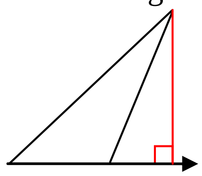
Altitudes for acute triangles lie inside the triangle.



Altitudes for right triangles lie on a side of the triangle.



Altitudes for obtuse triangle lie outside the triangle.



# Lesson 5-3: Concurrent Lines, Medians and Altitudes

Assign homework
p. 259  #1-15, 19-22, 27-29, 37-39